Continuous Equivalent Source Surface Approach for Accurate Interpolation and Continuation

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Abstract

Continuous Equivalent Source approach is presented to improve 3D potential field data interpolation and continuation. Most potential-field surveys are carried out on irregular grids or wide line spacings that do not allow convenient processing and inversion. Gridding, interpolation and reduction to a horizontal plane are required for high-resolution inversion in mine and oil-gas exploration. Continuous equivalent source surface could provide more accurate and stable solutions and clear images. This method is described and illustrated on several examples using of synthetic and field data (aeromagnetic, ground magnetic, and gravity).

Introduction

Gravity, magnetic, and aeromagnetic fields are often surveyed on rugged surfaces and irregular grids. The first thing of potential field work is to do gridding and reducing observed surface into a horizontal plane before inversion and quantitative analysis.
Now the popular gridding methods for 3-D gravity and magnetic data are Minimum-Curvature Surface (Smith and Wessel, 1990), Equivalent-Source (ES) method (Cordell, 1992), and Natural Neighbor method (Sambridge et al, 1995). However, the Minimum-Curvature Surface and Natural Neighbor method do not satisfy the potential field equations. The advantage of the Equivalent-Source Method is its basic physical idea of potential field being interpolated. The ES method thus has much in common with inversion; however, unfortunately, it also inherits from inversion some problems of performance and stability. Improvements of these aspects of the ES methods are the objectives of the study presented below.

Specifically, this research focuses on two aspects of ES inversion:

1. Improvement of the accuracy of gridding and interpolation;
2. Reduction of the conditional number of coefficient matrix for improved stability of the inversion.

The new method is referred below as the Continuous Equivalent Source (CES) method. The key difference of CES from other potential field interpolation techniques are that it uses a reformulation of the mathematical model to perform the two tasks above.

Method

The problem can be described as operator equation:

$$L(\mathbf{J}) = \Phi , \tag{1}$$

where Φ is the observed field (such as the gravity anomaly), **J** is the equivalent source field (e.g., the density) and L is the corresponding integral operator (in this case, convolution with the Green's function). By choosing a suitable weight function w, the basic equation of the Moment-Method is obtained:

$$\int_v w_i L(\widetilde{\mathbf{J}}) dv = \int_v w_i \Phi dv \tag{2}$$

The unknown function **J** can be expanded in a series using a family of linearly independent basis functions, N_i:

$$\widetilde{\mathbf{J}} = \sum_{j=1}^{n} N_j J_j \tag{3}$$

Note that the convergence and stability of the solution, as well as the computational efficiency, are determined by the choice of these basis functions. With approximation (3) for **J**, Eq.(2) becomes a linear system of equations:

$$\sum_{j=1}^{n} J_j \int_v w_i L(N_j) dv = \int_v w_i \Phi_i dv \, . \tag{4}$$

The resulting system of linear equations (4) is solved using the Least Squares method.

The heart of the CES method is to construct equivalent source surfaces that are continuous to at least the first order. Note that in Dampney's (1969) and Cordell's (1992) Discrete ES (DES) approaches are based on the choice of delta functions as N_i, (cf. Eq. 1):

$$\Phi(x,y,z) = \sum_{j=1}^{n} J(x_j^{'}, y_j^{'}, z_j^{'}) \cdot K(x - x_j^{'}, y - y_j^{'}, z - z_j^{'}) \, , \tag{5}$$

resulting in high gradients in the modeled fields and leading to instabilities in inversion. With a discrete set of basis functions, it is difficult to provide an accurate interpolation and gridding solution because of its sensitivity on the depth at which the effective sources are placed. When the ES points are too shallow, the model contains high gradients and the match with the data could be unstable; with deeper sources, the model becomes not sensitive to the gradients in the data. However, the CES approach overcomes these problems by using continuous N_i surfaces; with the wavenumber spectrum corresponding to that of the data. Unlike the DES method, CES uses continuous basis functions to express the ES surface by local integration and:

$$\{L(N_i)\}(x,y,z) = \iint_{\zeta,\eta} N_i(\xi,\eta,\zeta) \cdot K(x-\xi, y-\eta, z-\zeta) d\xi d\eta \, , \tag{6}$$

followed by summation (Eq. 4). Because spatially localized basis functions are used, only the values of the source functions at its corner nodes are used in order to interpolate the field inside a grid cell.

As a result of a careful choice of the set of basis functions (3), the coefficient matrix (4) typically acquires lower conditional number, which is of a benefit to the inversion.

Example

Several field data examples have been processed using this approach. As an example we consider a magnetic model with 3three bodies (Bhattacharyya ,1977) demonstrates the accuracy of this approach. The survey grid is equally spaced, with 27 X 27 nodes at intervals of 2 units of length. The peak topography on the survey surface is -5.3 unit; the valley is 1.7 unit. The field source is 3 cuboid bodies, with their parameters shown in the table below.

Tab. 1

	Coordinate of the cuboid						J (10 A/M)	I	D
	X1	X2	Y1	Y2	Z1	Z2			
I	-8	-3	-4	8	2	7	17000	30 N	45 W
II	0	5	0	7	3	7	34000	72 N	5 E
III	4	8	-9	-3	2	7	2000	60 N	10 E

In this model, the surface readings are upward and downward continued to two planes corresponding to the topographic high and low using the CES algorithm of this paper. The results are as follows:

In continuation into the -5.3 unit plane, the modeling accuracy is excellent, with theoretical amplitude is 858 nT, with a RMS error is 1.7 nT. Continuing to to the -2.0 unit plane, in most areas of downward continuation, the consistency between the modeled and observed fields is good, with amplitude of ~2000 nT, the RMS error of 7 nT. Both fields clearly reflect the presence of the 3 subsurface source bodies..

Conclusions

Continuous Equivalent Source (CES) method based on local basis functions is presented and applied to potential field data interpolation and continuation. The CES method offers strong advantages compared to the traditional interpolation methods in terms of accuracy and in the improvement of the conditional number of the coefficient matrix and consequently, stability of the inversion.

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References

Bhattacharyya, B.K.,and Chan, K.C.,1977, Reduction of magnetic and gravity data on arbitrary surfaces acquired in a region of high topographic relief: Geophysics, **42**, 1411-1430.

Cordell,L.E., and Grauch,V.,1985, Mapping basement magnetization zones from aeromagnetic data in the San Juan Basin, New Mexico, in Hinze, W.J., The unility of regional gravity and magnetic anomaly maps: Soc. Expl. Geophys., 181-197.

Cordell,L.E.,1992, A scattered equivalent-source method for interpolation and gridding of potential-field data in three dimensions, GEOPHYSICS, **57**, 4, 629-636.

Dampney, C.N.G., 1969, The equivalent source technique: Geophysics, **34**, 39-53.

Ducruix,J., LeMouel,J.L., and Courtillot,V., 1974, Continuation of three-dimensional potential field measured on an uneven surface: Geophys. J. of the Roy. Astr. Soc.,**38**,299-314.

Emilia, D.A., 1973, Equivalent source used as an analytic base for processing total magnetic field profiles: Geophysics, **38**, 339-348.

Guspi, F., 1987, Frequency-domain reduction of potential field measurements to a horizontal plane: Geoexp*l*., **24**, 87-98.

Handerson, R.G., and Cordell, L., 1971, reduction of unevenly spaced potential field data to a horizontal plane by means of finite harmonic series: Geophysics,**36**, 856-866.

Hansen, R.O., and Miyazaki,Y., 1984, Continuation of potential fields between arbitrary surfaces: Geophysic*s*,49,787-795.

Li, Yiong and H-J Gotze, 1999, Camparison of some griding methods, THE LEADING EDGE, August, 898- 900.

Nakatsuka, T., 1981, Reduction of magnetic anomalies to and from an arbitary surface: Geophysical Exploration, **34**, No.5, 6-13.

Parker, R.L., and Huestis, S.P., 1974, The inversion of magnetic anomalies in the presence of topography*: J. Geophys. Res.,***79**, 1587-1593.

Pilkington, M., 1989, Variable depth magnetization mapping: application to the Athabasca basin, northern Alberta and Saskatchewan, Canada: Geophysics, **54**, 1164-1173.

Pilkington, M., and Urquhart, W.E.S., 1990, Reduction of potential field data to a horizontal plane: Geophysics, **55**,549-555.

Sambridge,M., Braun,J. and McQueen,H.,1995, Geophysical parametrization and interpolation of irregular data using natural neighbours, Geophys. J. Int., **122**, 837-857.

Smith W.H.F. and Wessel P., 1990, Gridding with continuous curvature splines in tension, Geophysics, **55**, 291-305.

Strakhov, V. N., and Devitsyn, V. M., 1965, The reduction of observed values of a potential field to values at a constant level: Akad. Nauk USSR Izv. Fizika Zemli, 256-261.

Syberg, F.J.R., 1972, Potential continuation between general surfaces: Geophys.Prosp., **20**, 267-282.

Tsirulskiy, A.V., 1965, The reduction of observed potential fields to a single level: Akad. Nauk USSR Izv. Fizika Zemli, 190-193.

Xia, J., and Sprowl, D.R., 1991, Correction of topographic distortion in gravity data: Geophysics, **56**, 537-541.